MthSc 440, H440, 640 – Linear Programming

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Excerpt from the Syllabus

Homework:	25%	
Hour Tests (2):	40%	
Final exam:	35%	(May 3)

Learning and practice materials:

- **Class Notes**: MTHSC 440, *Linear Programming*, Campus Copy Shop.
- **Supplementary Handouts**: formulation and computational exercises

Course Outline

- Formulations
- Linear Programming models
- Optimality conditions
- The Simplex algorithm
- Refinements
- Duality theory
- KKT conditions
- Sensitivity analysis
- Dual Simplex algorithm
- Network flow models

Optimization models

- are used to find the best configuration of processes, systems, products, etc.
- rely on a theory developed primarily in the past 50 years
- have been applied to many industrial, financial, biological, and military problems:
    - refining processes
    - crew scheduling
    - forest management
    - law enforcement
- yield a more efficient use of budget/resources or a higher revenue

Success stories

Source: http://www.informs.com
(see also http://www.ScienceOfBetter.org)

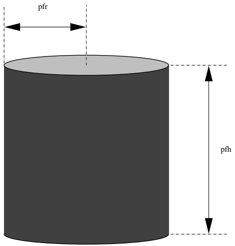
Year	Company	Resulta
86	Eletrobras (hydroelectric energy)	43M$ saved
90	Taco Bell (human resources)	7.6M$ saved
92	Harris semicond. prod. planning	50% → 95% orders “on time”
95	GM – Car Rental	+50M$
96	HP printers — re-designed prod.	2x production
99	IBM — supply chain	750M$ saved
00	Syngenta — corn production	5M$ saved
01	Ford — vehicle prototypes	250M$ saved

A simple example

- You work at a company that sells food in tin cans and are charged with designing the next generation can, which is a cylinder made of tin.
- The can must contain $V = 20$ cu. inches of liquid.
- Cut and solder tin to produce cylindrical cans.
- Tin is expensive, so we want to use as little as possible.

$\Rightarrow$ Design a cylinder with volume V using as little tin (i.e., total area) as possible.

Example



If we knew radius r and height h,

- the volume would be $\pi r^2 h$
- qty of tin would be $2\pi r^2 + 2\pi rh$

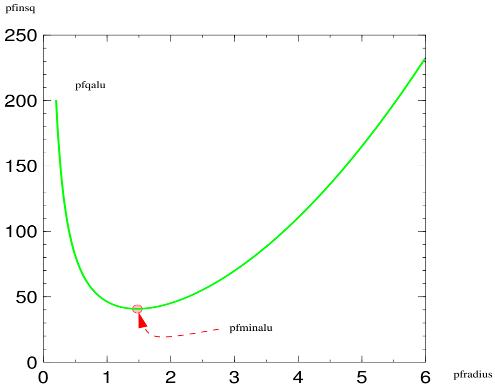
$\pi r^2 h$ must be $V = 20\text{in}^3 \Rightarrow h = \frac{V}{\pi r^2}$

Rewrite the quantity of tin as $Q(r) = 2\pi r^2 + 2\pi r \frac{V}{\pi r^2}$, or

$$Q(r) = 2\pi r^2 + \frac{2V}{r}$$

$\Rightarrow$ Find the minimum of $Q(r)$ — a calculus problem!

Minimize the quantity of tin



$r = \boxed{\mathbf{1.471\ in}}$ $\quad h = \frac{V}{\pi(1.471)^2} = 2.942 \text{ in}$

Your first optimization model

Variables	r: radius of the can's base
	h: height of the can
Objective	$2\pi rh + 2\pi r^2$ (minimize)
Constraints	$\pi r^2 h = V$
	$h \geq 0$
	$r \geq 0$

Optimization models, in general, have

Variables: Height and radius, number of trucks, etc.
— the *unknowns* of the problem.

Constraints: Physical, explicit ($V = 20\text{in}^3$), imposed by physical laws, budget limits, ...
They define **all** and the **only** values of the variables that give possible solutions.

Objective function: what the boss really cares about.
Quantity of tin, total cost of trucks, total estimated revenue, ... — a function of the variables.

The general optimization problem

Consider a vector $x \in \mathbb{R}^n$ of variables.

An optimization problem can be expressed as:

$$\mathbf{P}: \quad \begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_1(x) \leq b_1 \\ & f_2(x) = b_2 \\ & \quad \vdots \\ & f_m(x) \geq b_m \end{array}$$

In this class we concentrate on linear optimization problems (*linear programs*).